

# RADIACIÓN NUCLEAR

Ley de decaimiento radioactivo:

$$\frac{dN}{dt} = -\lambda \cdot N$$

$\lambda$ : **probabilidad de decaimiento por unidad de tiempo**

$$N = N_0 \cdot e^{-\lambda t}$$

$\tau_m = \frac{1}{\lambda}$  tiempo de vida medio (mean lifetime)

$$\frac{N}{N_0} = \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$T_{1/2} = \frac{1}{\lambda} \cdot \ln 2$  tiempo de vida mitad (half lifetime)

Ejemplo:

Cs<sup>137</sup>  $T_{1/2} = 27$  años  $\longrightarrow$

$\lambda$  (por núcleo) =  $8,2 \cdot 10^{-10}$  1/seg.

1  $\mu$ g de Cs:  $10^{15}$  nucleos  $\longrightarrow$

Actividad (intensidad):  $8 \cdot 10^5$  1/seg (Bq)

1Ci =  $3,7 \cdot 10^{10}$  Bq

Distribución de Poisson:

$$P(r) = \frac{\mu^r \cdot e^{-\mu}}{r!} \quad \mu = N \cdot p \quad \sigma^2 = Np(1-p)$$

$$\mu = \lambda \cdot t \rightarrow P(r, t) = \frac{(\lambda t)^r \cdot e^{-\lambda t}}{r!}$$

Ej:

probabilidad de un evento en  $t=T+dt$

$$P(t) = \lambda \cdot e^{-\lambda t}$$

## DETECTORES DE RADIACIÓN

TABLE 65.1 Characteristics of Nuclear Radiations [8]

Type	Origin	Process	Charge	Mass [MeV]	Spectrum (energy)
$\alpha$ -particles	Nucleus	Nuclear decay or reaction	+2	3727.33	Discrete [MeV]
$\beta^-$ -rays (negatrons)	Nucleus	Nuclear decay	-1	0.511	Continuous [keV-MeV]
$\beta^+$ -rays (positrons)	Nuclear	Nuclear decay	+1	0.511	Continuous [keV-MeV]
$\gamma$ -rays	Nucleus	Nuclear deexcitation	0	0	Discrete [keV-MeV]
X-rays	Orbital electrons	Atomic deexcitation	0	0	Discrete [eV-keV]
Internal conversion electrons	Orbital electrons	Nuclear deexcitation	-1	0.511	Discrete [high keV]
Auger electrons	Orbital electrons	Atomic deexcitation	-1	0.511	Discrete [eV-keV]
Neutrons	Nucleus	Nuclear reaction	0	939.57	Continuous or discrete [keV-MeV]
Fission fragments	Nucleus	Fission	$\cong 20$	80-160	Continuous (bimodal) 30-150 MeV

## INTERACCIÓN DE LA RADIACIÓN CON LA MATERIA.

### PARTÍCULAS CARGADAS:

- Colisiones inelásticas.
- Dispersión elástica (por núcleos).
- Radiación Cherenkov.
- Radiación de frenado (Bremsstrahlung).
- Reacciones nucleares.

### RADIACIÓN $\gamma$ , RAYOS X:

- Efecto fotoeléctrico.
- Dispersión Compton.
- Producción de pares.

## DETECTORES DE RADIACIÓN.

Parámetros de importancia:

EFICIENCIA:

- Eficiencia absoluta:  $\frac{\text{eventos registrados}}{\text{eventos totales emitidos por la fuente}}$
- Eficiencia intrínseca:  $\frac{\text{eventos registrados}}{\text{eventos que llegan al detector}}$

SENSIBILIDAD

- Masa del volumen sensible.
- Sección eficaz de la interacción.
- Ruido (fluctuaciones).
- Características constructivas y geometría.

RESPUESTA

Proporcional a la energía incidente.

- La radiación deposita TODA su energía en el detector.  
(un caso particular: cámara de ionización).
- Independencia de la posición de la ionización primaria.

RESOLUCIÓN (R)

$$R = \frac{\Delta E}{E} \quad \text{el ancho (relativo) a mitad de altura (FWHM)}$$

Depende del detector (tipo de interacción)

p. ej    R= 9%            para un cristal de NaI.  
          R= 0,1%        para un cristal de Ge.

siendo  $J = \frac{E}{w}$

E= energía de la radiación incidente

w= energía necesaria para producir 1 ionización.

Suponiendo que  $\sigma^2 = J$  (Poisson)

$$R = \frac{2,35\sqrt{J}}{J} = 2,35\sqrt{\frac{w}{E}}$$

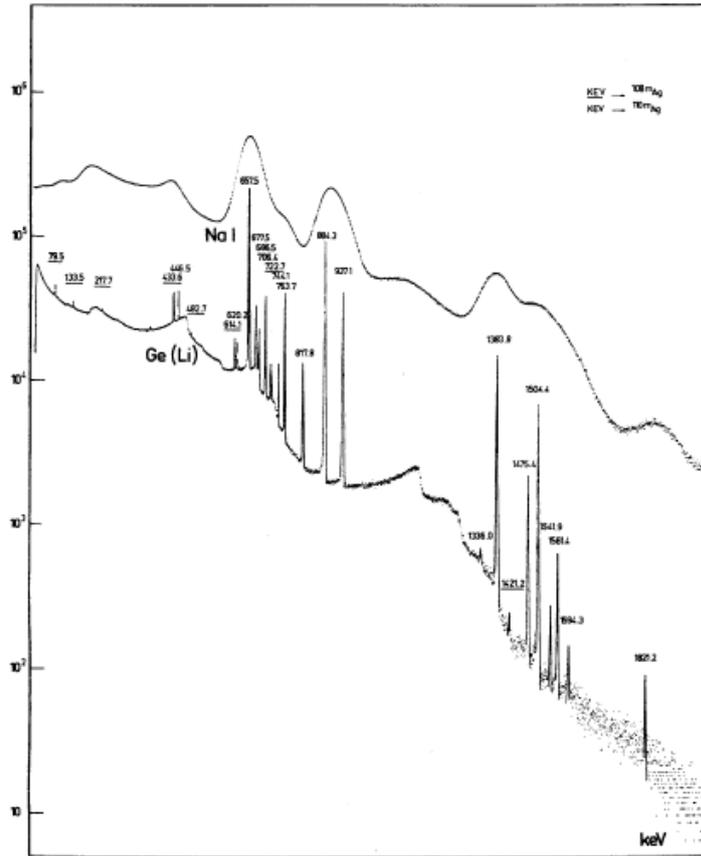
pero en general  $\sigma^2 = F \cdot J$  (F: Fano factor)  
y

$$R = 2,35\sqrt{\frac{F \cdot w}{E}}$$

La resolución total del sistema de detección será:

$$(\Delta E_{total})^2 = (\Delta E_{detector})^2 + (\Delta E_{electrónica})^2 + \dots +$$

# Resolution of NaI(Tl) and Ge detectors



(J.C.I. Philippot, IEEE Trans. Nucl. Sci. NS-17/3 (1970) 446)

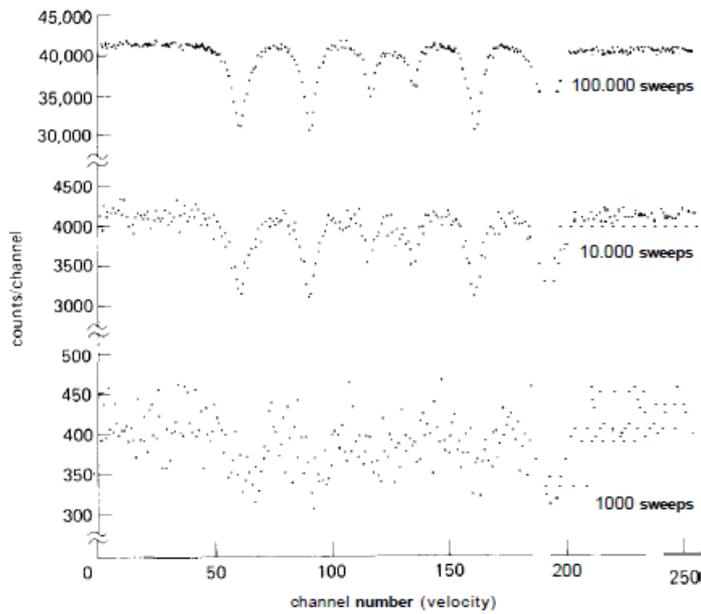
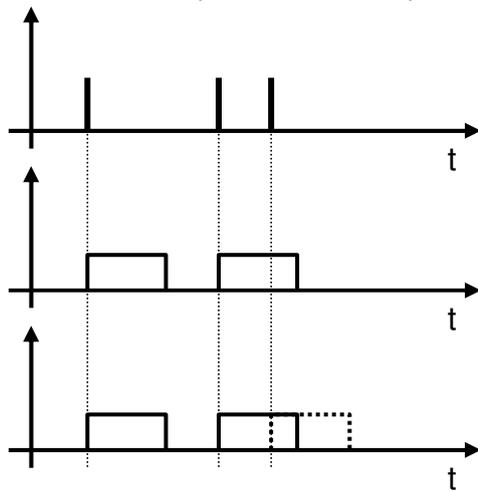


Figure 15.35. Mossbauer absorption spectrum, showing effect of signal averaging.

# TIEMPO MUERTO (DEAD TIME)



no extensible

extensible o paralizabile

$n$ : eventos producidos/segundo

$k$ : eventos registrados en un período  $T$

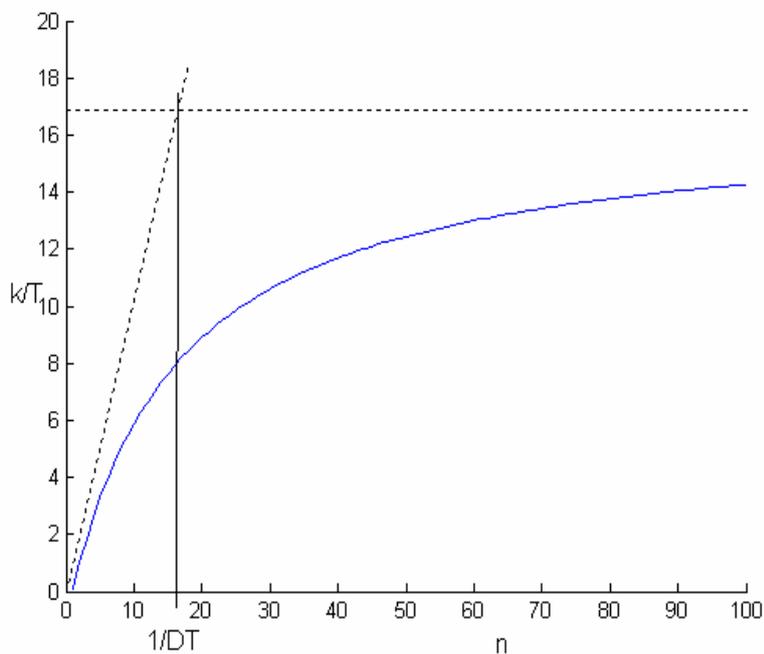
$DT$ : tiempo muerto del sistema

Sistema no extensible:

eventos perdidos  
eventos totales (en  $T$ )

$$\frac{nk \cdot DT}{nT} = \frac{k + nk \cdot DT}{nT}$$

$$\frac{k}{T} = \frac{n}{1 + n \cdot DT}$$



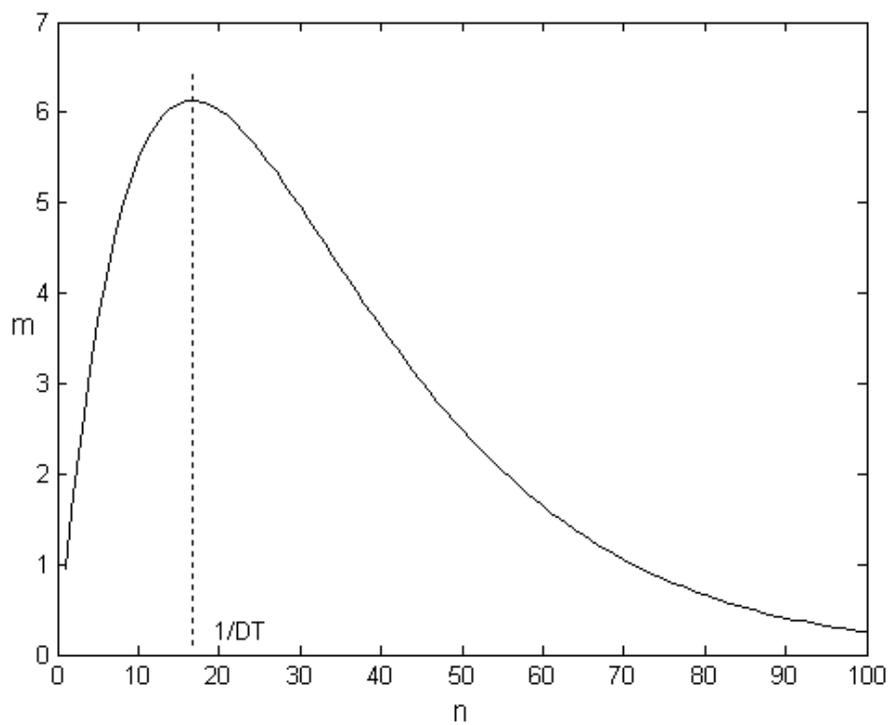
Sistema extendible:

probabilidad de un evento en  $dt$ , después de  $t$

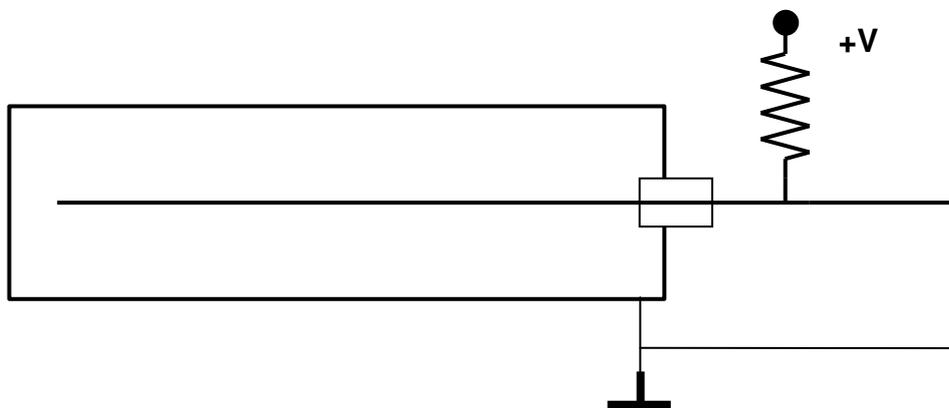
$$P(t) = \frac{(nt)^0 e^{-nt}}{0!} \cdot n dt = n \cdot e^{-nt} dt$$

$$P(DT) = \int_{DT}^{\infty} n e^{-nt} dt = e^{-nDT}$$

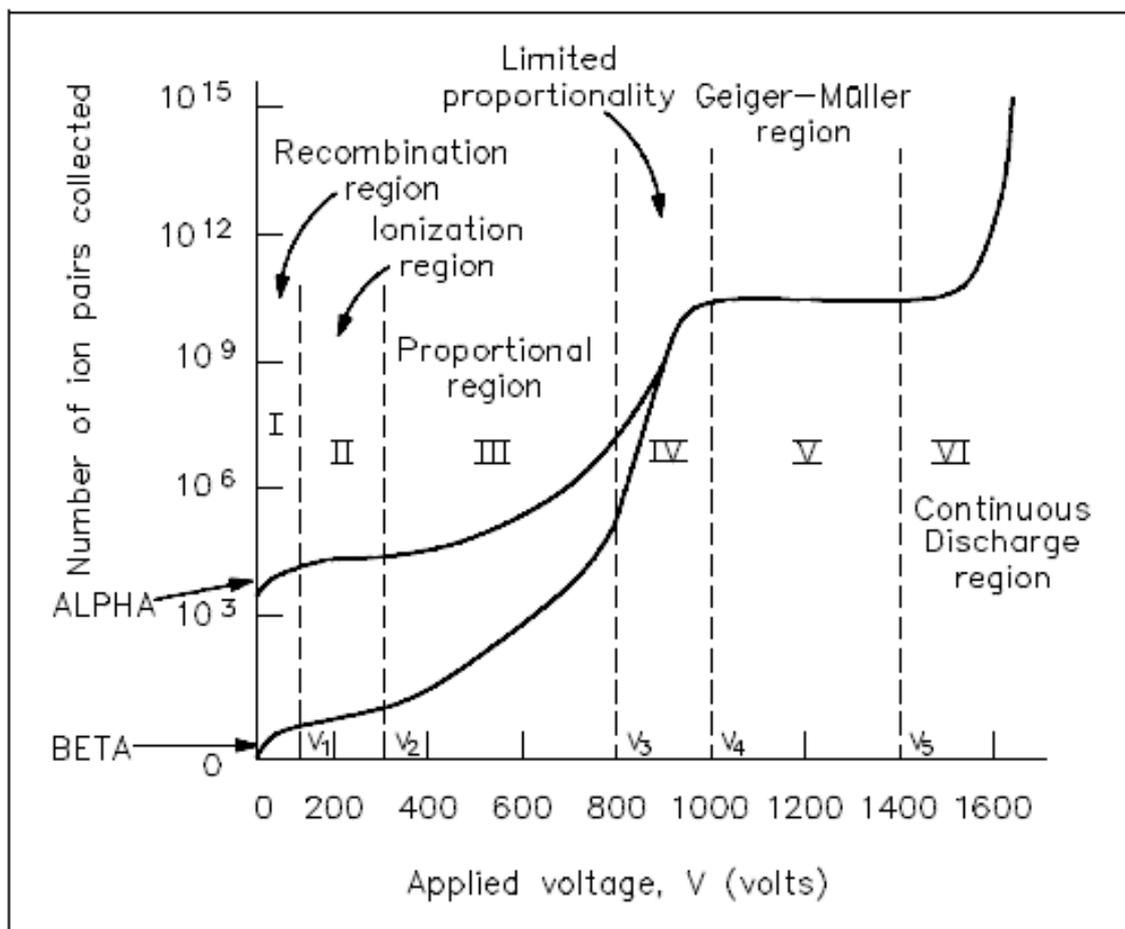
$$k = n e^{-nDT}$$



# DETECTORES GASEOSOS:



$$E = \frac{1}{r} \cdot \frac{V}{\ln\left(\frac{b}{a}\right)}$$



## EVOLUCIÓN TEMPORAL DE LA SEÑAL EN UN DETECTOR PROPORCIONAL.

$$E = \frac{1}{r} \cdot \frac{V}{\ln\left(\frac{b}{a}\right)}$$

$$C = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)}$$

$$E(r) = \frac{1}{r} \cdot \frac{V \cdot C}{2\pi\epsilon} \Rightarrow \Phi(r) = \frac{V \cdot C}{2\pi\epsilon} \cdot \ln\left(\frac{r}{a}\right)$$

$$W_q = q \cdot \Phi$$

$$W_c = \frac{1}{2} C V^2 \cdot l$$

$$v_{drift} = \frac{\partial r}{\partial t} = \mu \cdot E(r)$$

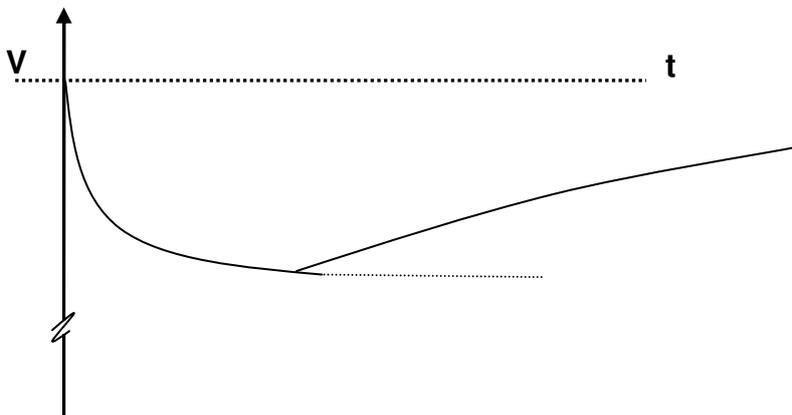
$$C \cdot V \cdot l \cdot dV = q \cdot \frac{\partial \Phi}{\partial r} dr$$

$$V(t) = \frac{q}{4\pi\epsilon l} \ln\left(1 + \frac{\mu \cdot C \cdot V}{\pi\epsilon a^2} \cdot t\right)$$

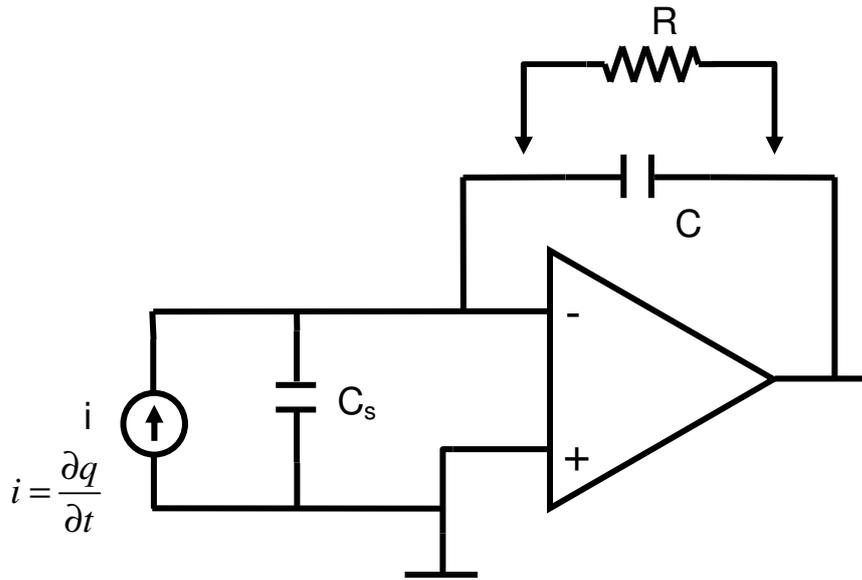
$$V(t) = \frac{q}{C} \cdot \frac{1}{\ln\left(\frac{b}{a}\right)} \cdot \ln\left(1 + \frac{2\mu V}{\ln\left(\frac{b}{a}\right) a^2} \cdot t\right)$$

haciendo  $\frac{\pi\epsilon a^2}{\mu C V} = t_0$

el tiempo de "drift" total resulta  $T = \frac{t_0}{a^2} \cdot (b^2 - a^2)$



# AMPLIFICADOR DE CARGA

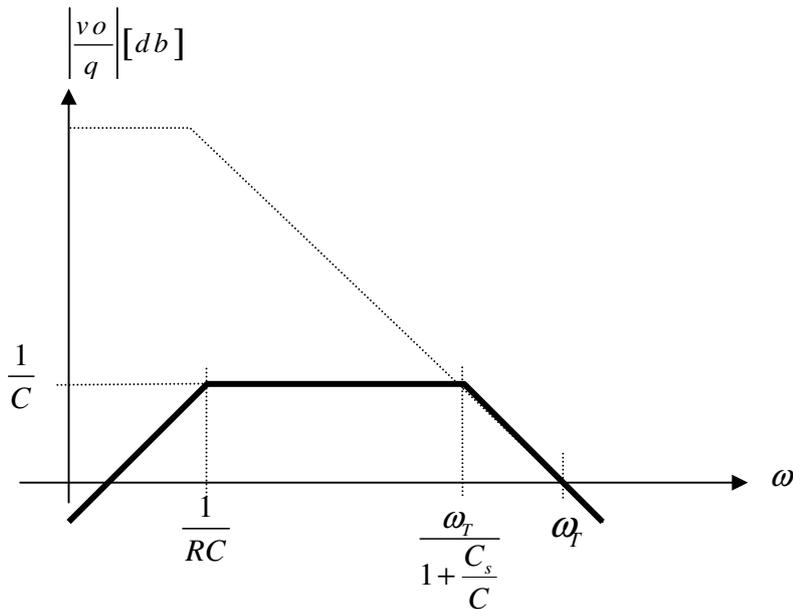


bajas frecuencias:

$$v_o = -s \cdot q \left( \frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}} \right) \Rightarrow \frac{v_o}{q} = -\frac{1}{C} \cdot \frac{sCR}{1+sCR}$$

En frecuencias altas:

$$\frac{v_o}{q} = -\frac{1}{C} \cdot \frac{1}{1+s \frac{1}{\omega_T} \frac{Cs}{1+\frac{Cs}{C}}}$$



# POLARIZACIÓN DEL FOTOMULTIPLICADOR

Table 5.1  
Gain and pulse linearity of a fast response photomultiplier,  
with three types of voltage distribution

Type of voltage distribution	Gain $V_{ht} = 2500 \text{ V}$	Linear within 2% for current pulse amplitudes up to
A	$1.2 \times 10^8$	40 mA
B	$0.7 \times 10^6$	250 mA
C	$2 \times 10^7$	100 mA

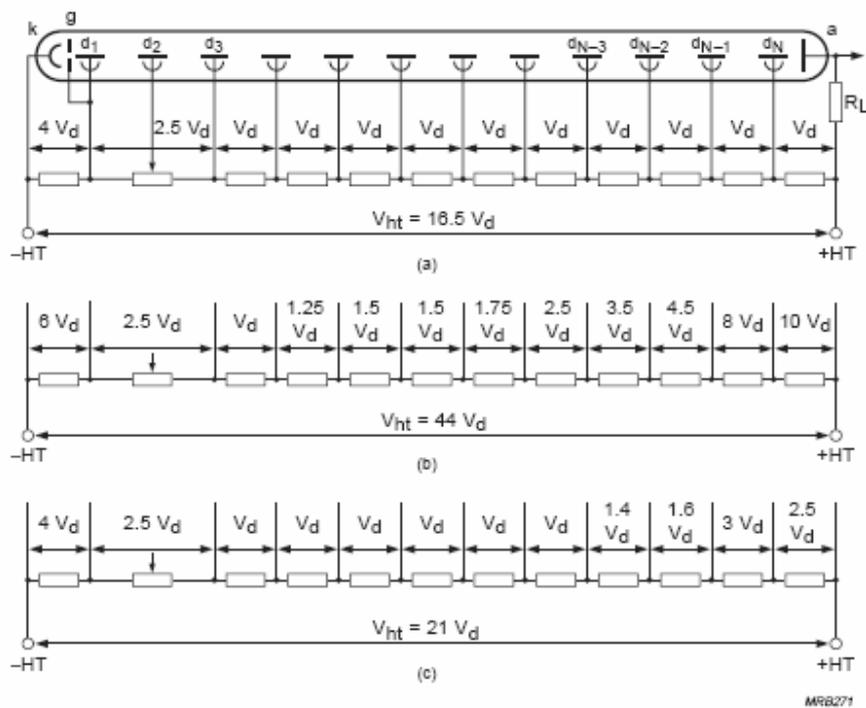
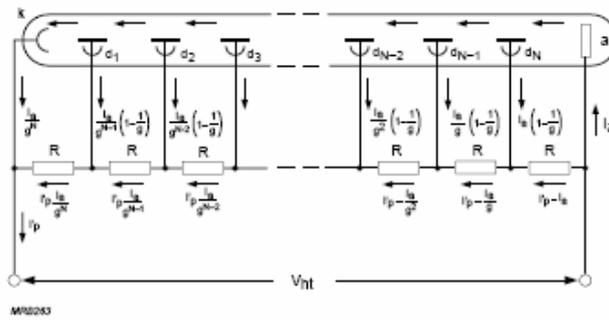


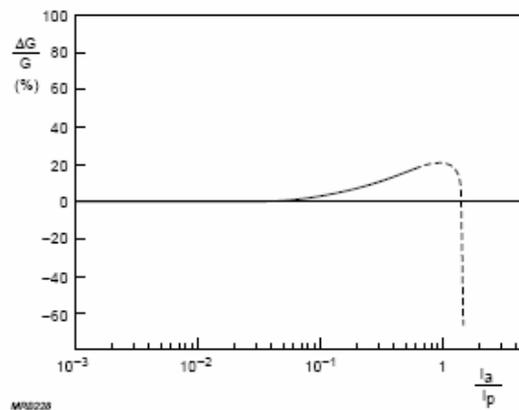
Fig.5.7 Voltage dividers: (a) type A equal step, (b) type B progressive, (c) type C intermediate

# ESTABILIDAD DE LA POLARIZACIÓN, LINEALIDAD



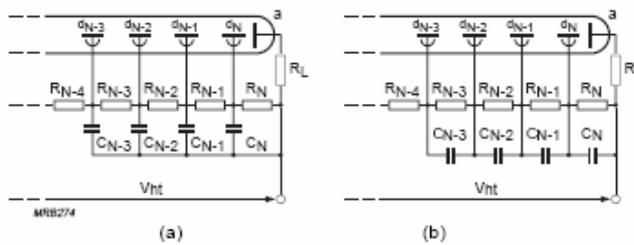
MRB283

Fig.4.11 Current distribution in an iterative voltage divider



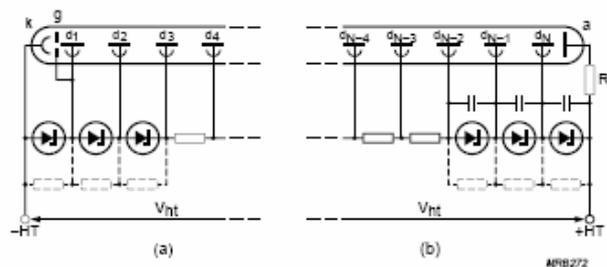
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Fig.4.12 Gain variation (overlinearity) as a function of the ratio  $I_a/I_p$



MRB274

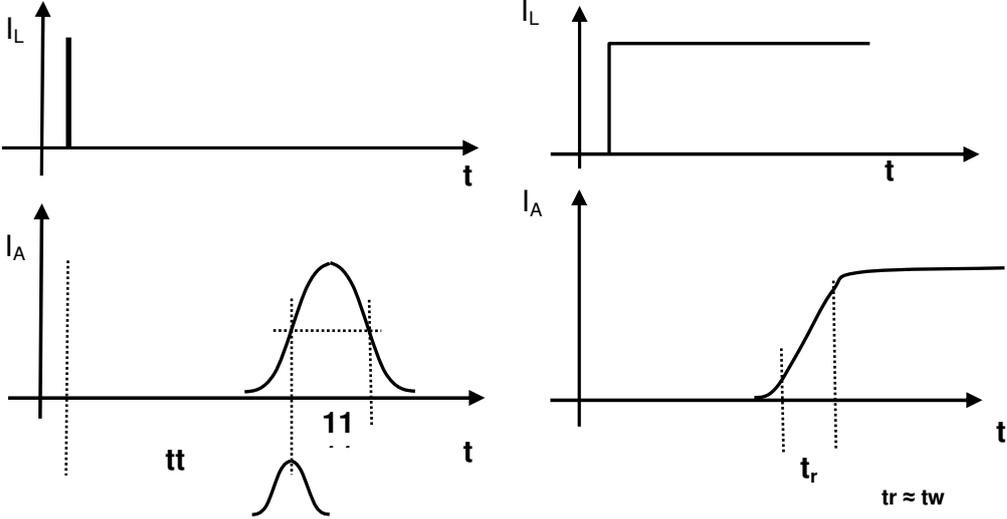
Fig.5.10 Dynode decoupling: (a) parallel, (b) series.



MRB272

Fig.5.8 Inter-electrode voltage stabilization with zener diodes: (a) in first stages, (b) in last stages (dashed lines, protection resistors)

PARÁMETROS DE IMPORTANCIA:



XP-2020Q  $t_w = 2,5 - 3$  nseg.  
 $tt = 30$  nseg.  
 $\sigma_{tt} = 250$  pseg.

# ACOPLAMIENTO DE LA SEÑAL DE SALIDA

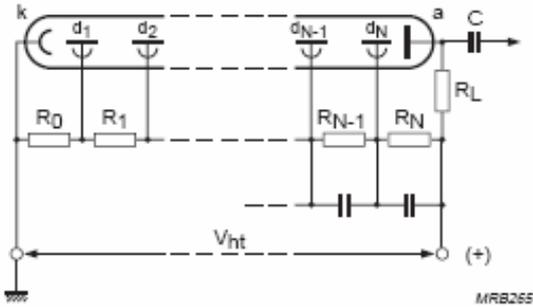


Fig.5.1 Positive-polarity voltage supply

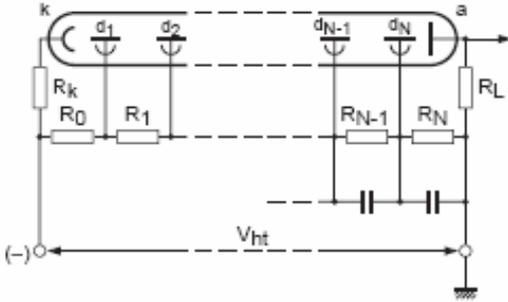
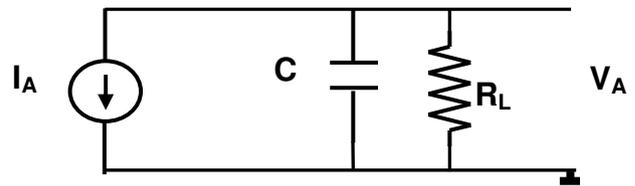
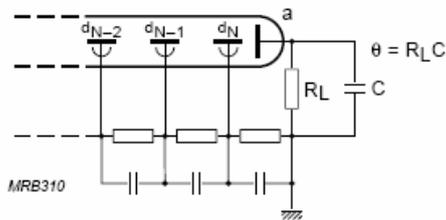


Fig.5.2 Negative-polarity voltage supply

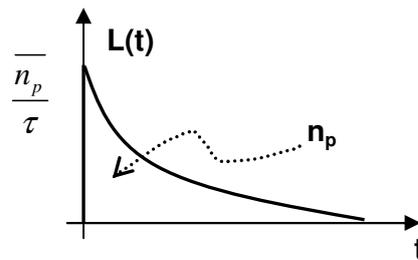
## EVOLUCIÓN TEMPORAL DEL PULSO DE ÁNODO

consideremos el siguiente modelo:



La luz emitida por el centelleador es de la forma:

$$L(t) = \left( \frac{\overline{n_p}}{\tau} \right) \cdot e^{-\frac{t}{\tau}}$$



si  $t_w \ll \tau$

$$I_A(t) = \left( \frac{q_A}{\tau} \right) \cdot e^{-\frac{t}{\tau}}$$

$$V_A(t) = \frac{q_A}{C} \cdot \frac{R_L C}{\tau - R_L C} \left( e^{-\frac{t}{\tau}} - e^{-\frac{t}{R_L C}} \right)$$

el valor máximo es:

$$V_{AP} = \frac{q}{C} \cdot \left( \frac{R_L C}{\tau} \right)^{\frac{1}{1 - \frac{R_L C}{\tau}}}$$

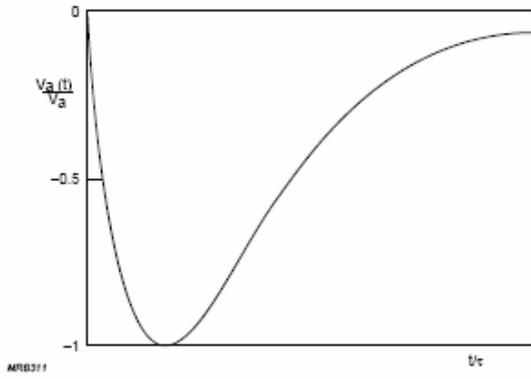


Fig.6.8 Voltage pulse shape over the anode load  $R_L C$

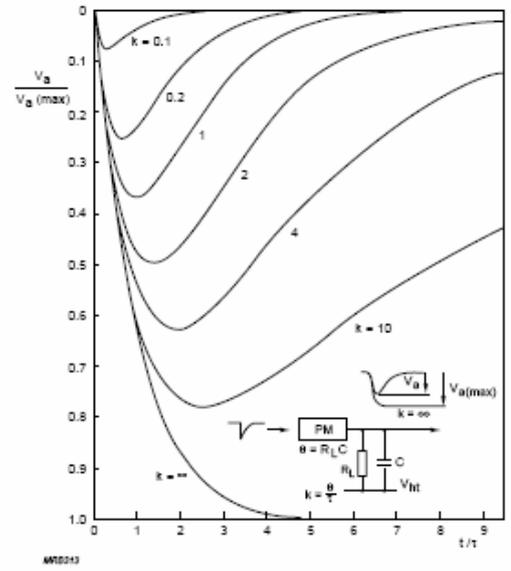
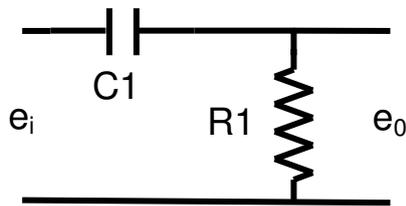
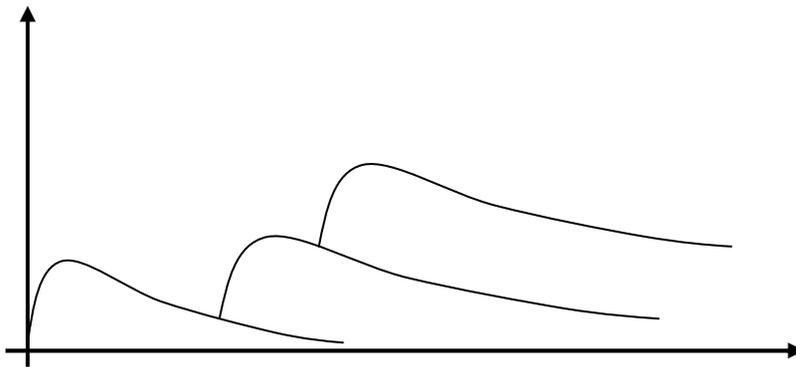
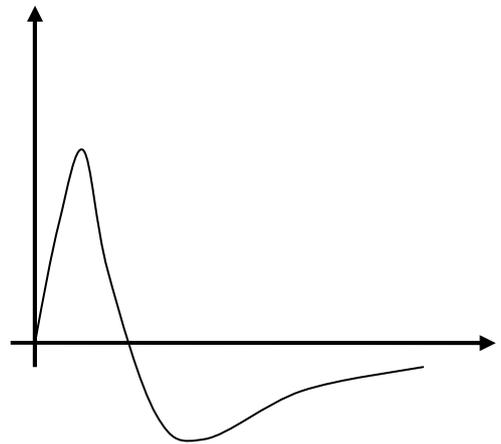
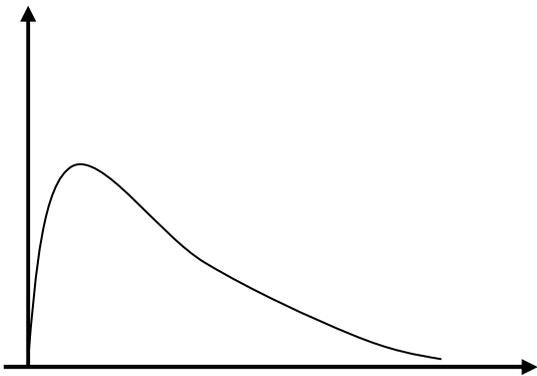


Fig. 6.10 Relative pulse amplitude shapes for an anode load  $R_L C$  with  $k = \theta/\tau$  as parameter

# COMPENSACIÓN POLO- CERO



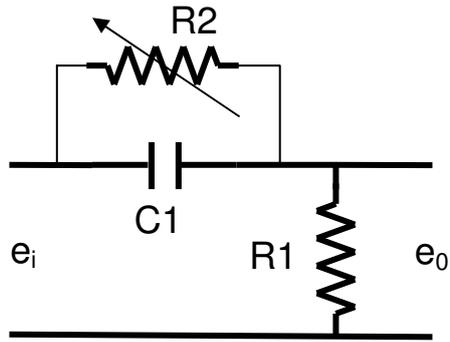
$$e(t) = E \cdot e^{-\frac{t}{T_0}} \Rightarrow e(s) = E \cdot \frac{1}{s + T_0}$$



$$G(s) = \frac{sC1R1}{1 + sC1R1} = \frac{s}{s + \frac{1}{T_1}} \quad T_1 = C1R1$$

$$e_0(s) = E \frac{1}{s + T_0} \cdot \frac{s}{s + T_1} \Rightarrow$$

$$\Rightarrow e_0(t) = \frac{E}{T_0 - T_1} \cdot \left[ T_1 \cdot e^{-\frac{t}{T_0}} - T_0 \cdot e^{-\frac{t}{T_1}} \right]$$



$$G(s) = \frac{s + \frac{1}{T_2}}{s + \frac{1}{T_3}} \quad T_2 = C_1 \cdot R_2; \quad T_3 = C_1 \left( \frac{R_1 \cdot R_2}{R_1 + R_2} \right)$$

$$e_0(s) = E \cdot \frac{1}{s + \frac{1}{T_0}} \cdot \frac{s + \frac{1}{T_2}}{s + \frac{1}{T_3}}$$

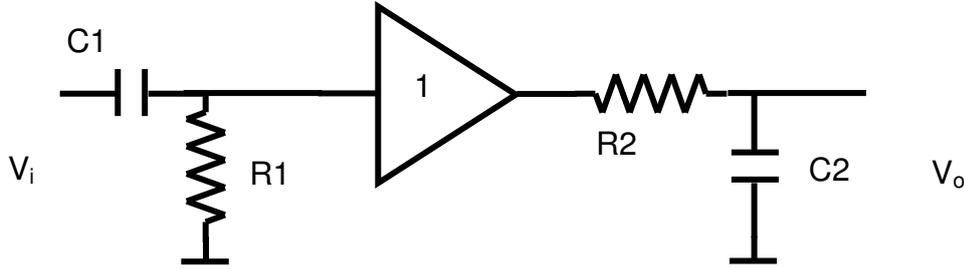
*haciendo :*

$$s + \frac{1}{T_2} = s + \frac{1}{T_0}$$

resulta:

$$e_0(t) = E \cdot e^{-\frac{t}{T_3}}$$

# SHAPING (FILTRADO)

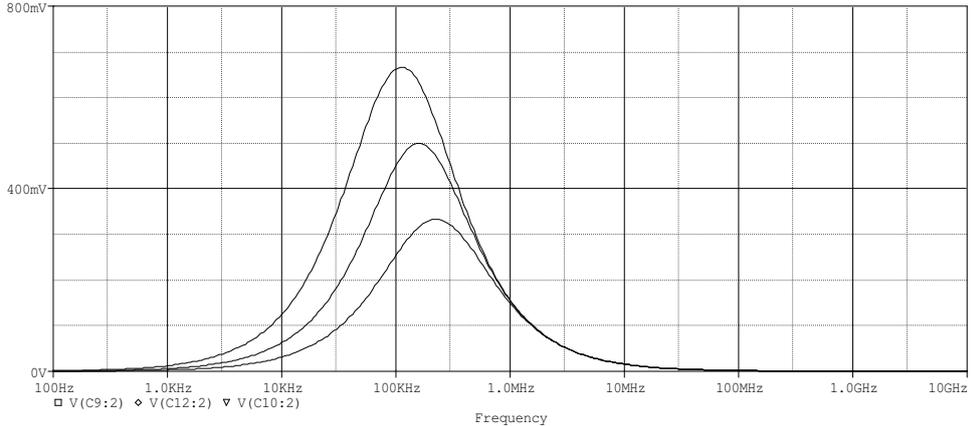
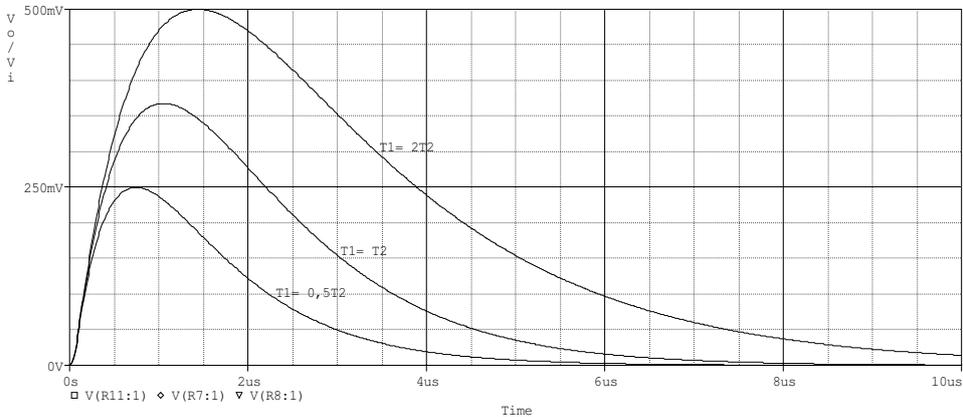


$$\frac{V_o(s)}{V_i(s)} = \frac{s}{1 + sC_1R_1} \cdot \frac{1}{1 + sC_2R_2}$$

$$T_1 = R_1C_1$$

$$T_2 = R_2C_2$$

$$V_o(t) = V_i \frac{T_1}{T_1 - T_2} \left( e^{-\frac{t}{T_1}} - e^{-\frac{t}{T_2}} \right)$$



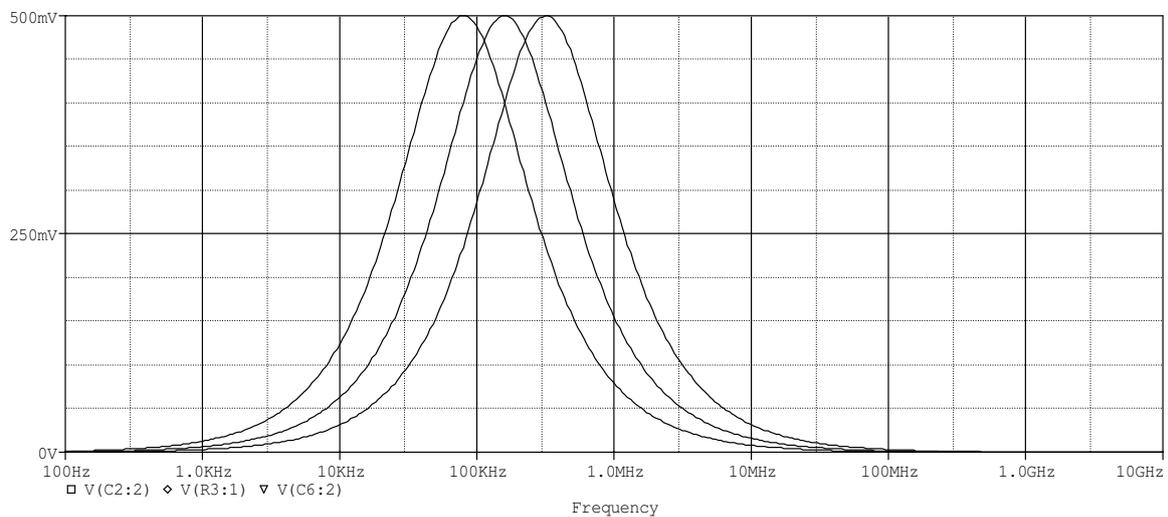
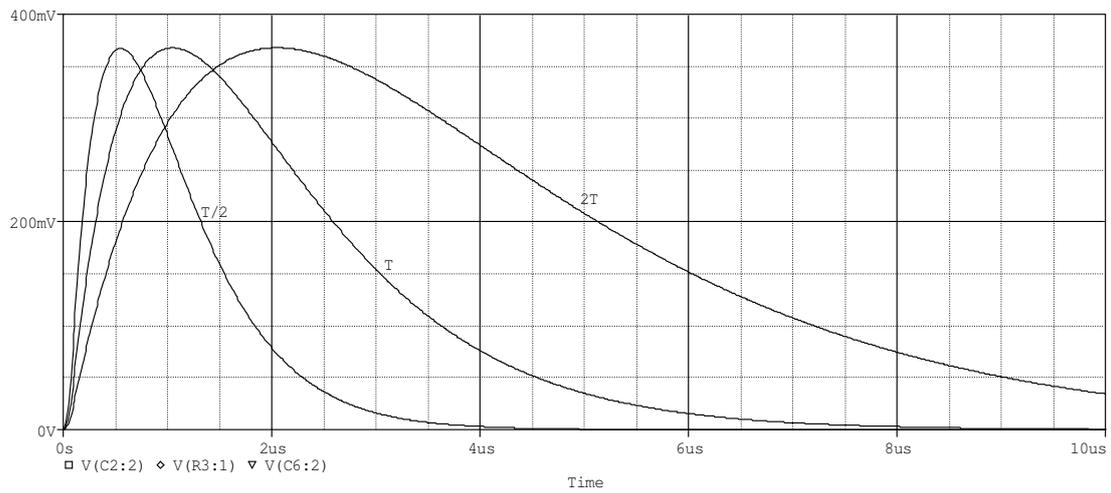
si  $T_1 = T_2 = T$

$$V_o(t) = V_i \frac{t}{T} e^{-\frac{t}{T}}$$

Tiempo al máximo (peaking time) =  $T$

Valor máximo =  $\frac{V_i}{e}$

Ancho al 10%  $\approx 5T$



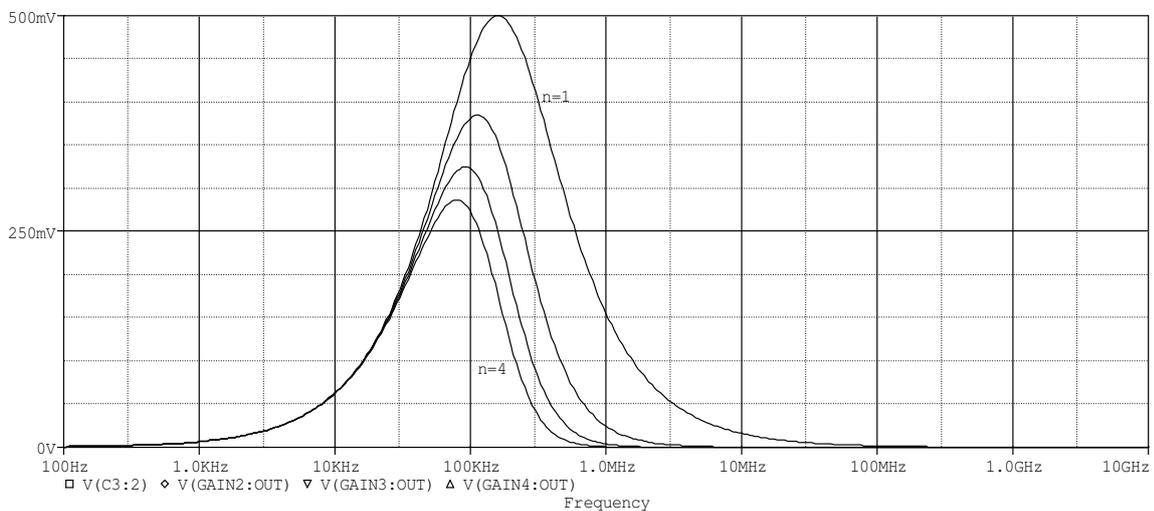
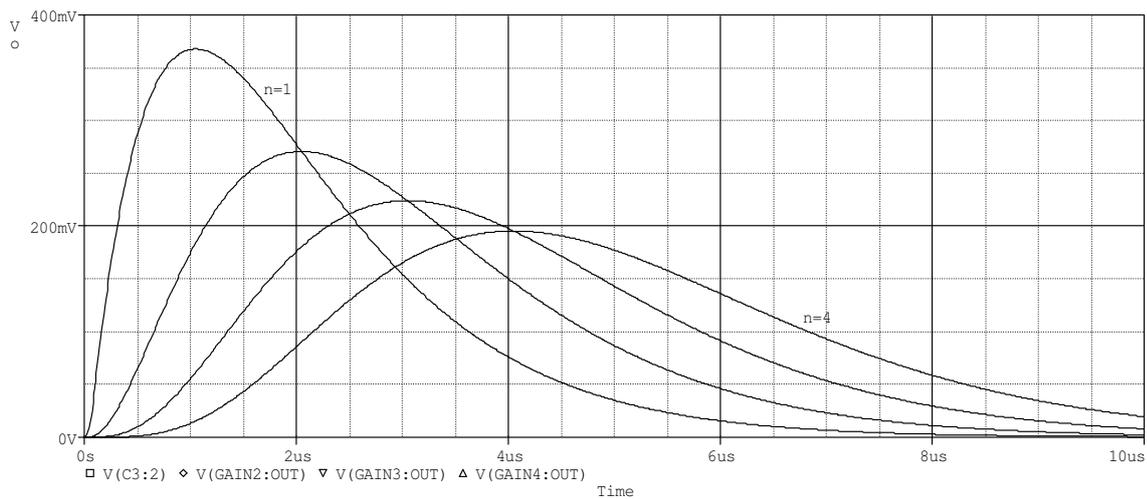
## APROXIMACIÓN AL FILTRO GAUSSIANO

$$\frac{V_o(s)}{V_i(s)} = \frac{s}{1 + sC_1R_1} \cdot \left( \frac{1}{1 + sC_2R_2} \right)^n$$

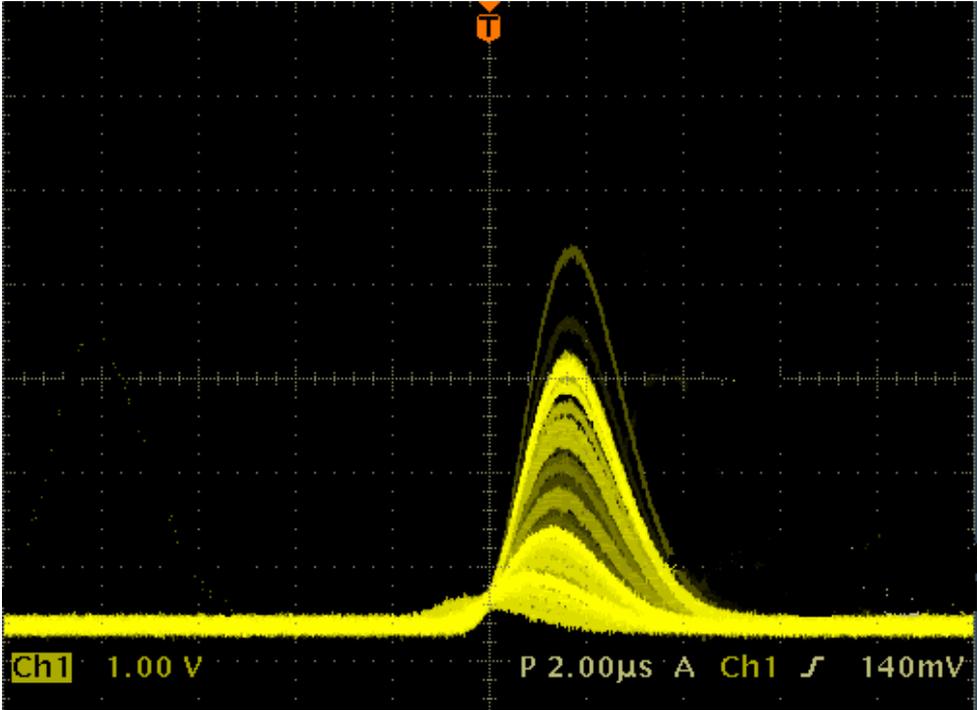
$$V_o(t) = \frac{V_i}{n!} \left( \frac{t}{T} \right)^n \cdot e^{-\frac{t}{T}}$$

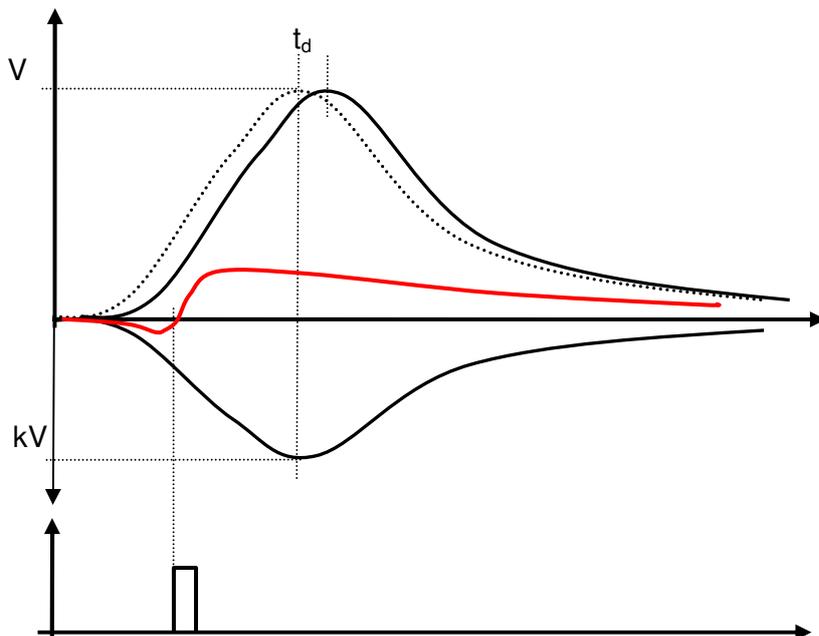
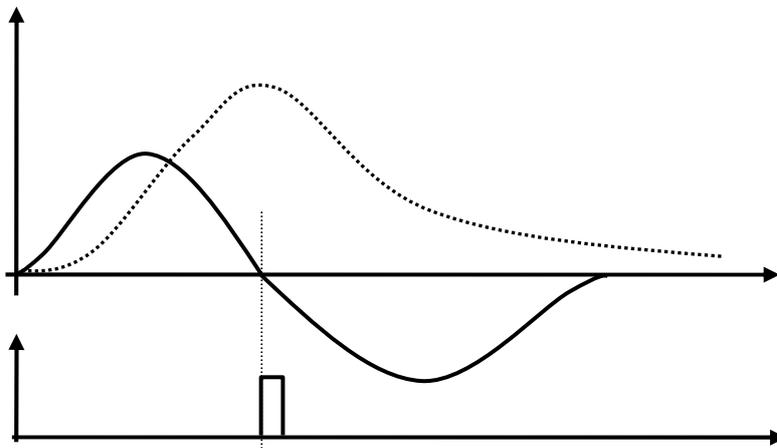
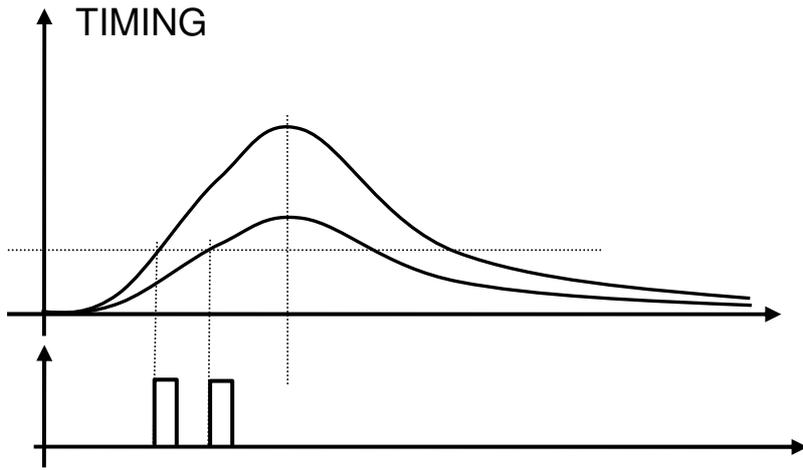
Tiempo al máximo (peaking time) =  $nT$

Valor máximo =  $V_i \frac{n^n \cdot e^{-n}}{n!}$

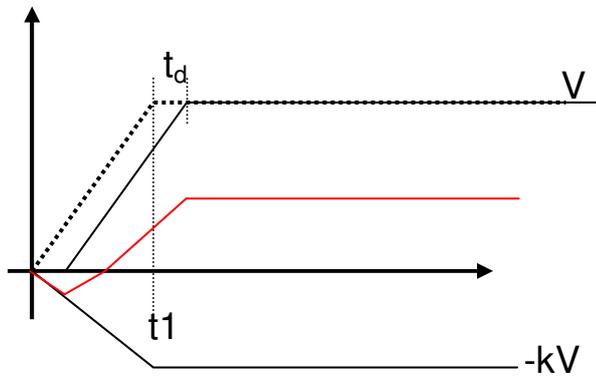


Ejemplo





Una demostración simple:



$$v = \frac{V}{t_1} t$$

$$\frac{V}{t_1} \cdot (t - t_d) - \frac{kV}{t_1} \cdot t = 0$$

$$\frac{V}{t_1} \cdot t \left( 1 - \frac{t_d}{t} - k \right) = 0$$

$$t = \frac{t_d}{1 - k}$$